

Manda turg assignment for October 23, 2009

MA1 2300

In all problems $f(z) = \frac{1}{1+z^2}$.

1. Let r be a real number, $r > 0$ and let L_r be the line from the point $-r$ to r in \mathbb{C} . Let γ_r be the upper half circle with radius r and center 0 . That is

$$\gamma_r = \{z \in \mathbb{C} \mid |z| = r \text{ \& } \text{Im } z \geq 0\}.$$

a) Compute $\int_{L_r} f(z) dz$.

b) Use Cauchy's integral theorem & a) to determine $\int_{\gamma_r} f(z) dz$ when $r < 1$.

c) Compute $\int_{\gamma_r} f(z) dz$ when $r > 1$.

2. a) Show that $f(z) = \frac{1}{2} \left[\frac{1}{1+iz} + \frac{1}{1-iz} \right]$.

b) Find the Taylor series for f around $z=0$.